Risk management strategies for banks

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Abstract

We analyze optimal risk management strategies of a bank financed with deposits and equity in a one period model. The bank's motivation for risk management comes from deposits which can lead to bank runs. In the event of such a run, liquidation costs arise. The hedging strategy that maximizes the value of equity is derived. We identify conditions under which well known results such as complete hedging, maximal speculation or irrelevance of the hedging decision are obtained. The initial debt ratio, the size of the liquidation costs, regulatory restrictions, the volatility of the risky asset and the spread between the riskless interest rate and the deposit rate are shown to be the important parameters that drive the bank's hedging decision. We further extend this basic model to include counterparty risk constraints on the forward contract used for hedging.

1. Introduction

The focus of this paper is to study the rationale for banks' risk management strategies where risk management is defined as set of hedging strategies to alter the probability distribution of the future value of the banks' assets.

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There is a broad literature on these decisions for firms in general, beginning with Modigliani and Miller (1959): Their famous theorem states that in a world of perfect and complete markets, financial decisions are irrelevant as they do not alter the value of the shareholder's stake in the firm. The only way to increase shareholder's wealth is to increase value of the firm's assets. Neither the capital structure nor the risk management decisions have an impact on shareholder's wealth.

Some important deviations from the perfect capital markets in the Modigliani–Miller setting have been identified, giving motivations for firms to care about risk management, such as taxes, bankruptcy costs, agency costs and others (Froot et al., 1993; Froot and Stein, 1998; Smith and Stulz, 1985; DeMarzo and Duffie, 1995; Stulz, 1996; Shapiro and Titman, 1986). When these reasons for risk management are incorporated into the firm's objective function, one finds the following basic result: When all risks are perfectly tradeable the firm maximizes shareholder value by hedging completely (Froot and Stein, 1998; Broll and Jaenicke, 2000; Mozumdar, 2001). [1]

However, the Modigliani–Miller-theorem as well as the aforementioned hedging motives are ex ante propositions: Once debt is in place, ex post financial decisions can alter the equity value by expropriating debt holders. This strategy is known as asset substitution (Jensen and Meckling, 1976). Because of limited liability, the position of equity holders can be considered as a call option on the firm value (Black and Scholes, 1973). This implies that taking on as much risk as possible is the optimal ex post risk management strategy. In summary, theory is inconclusive regarding the question of the optimal hedging strategy of firms.

Turning to the question of optimal hedging and capital structure decisions of banks, a first finding is that the analysis within the neoclassical context of the Modigliani–Miller-theorem would be logically inconsistent. Banks are redundant institutions in this case and would simply not exist (Freixas and Rochet, 1998, p. 8). The keys to the understanding of the role of banks and their financial decisions are transaction costs and asymmetric information. These features have been dealt with extensively in the banking literature, departing from the neoclassical framework (Baltensperger and Milde, 1987; Freixas and Rochet, 1998; Merton, 1995; Schrand and Unal, 1998; Bhattacharya and Thakor, 1993; Diamond, 1984, 1996; Kashyap et al., 2002; Allen and Santomero, 1998, 2001):

- Banks have illiquid or even nontradeable long term assets because of the transformation services they provide.
- Part of the illiquidity of banks' assets can be explained by their information sensitivity; banks can have comparative informational advantages due to their role as delegated monitors. Examples include information about bankruptcy probabilities and recovery rates in their credit portfolio. This proprietary information

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[1] This result is a consequence of the payoff-function's concavity induced by the risk management motives and the application of Jensen's inequality.

can be further improved through long term relationships with creditors (Boot, 2000; Diamond and Rajan, 2000).

- In contrast to other firms, banks' liabilities are not only a source of financing but rather an essential part of their business: Depositors pay implicit or explicit fees for deposit-related services (i.e. liquidity insurance, payment services, storage). The leverage in banks' balance sheets is thus many times higher.
- Bank deposits can be withdrawn at any time. The sequential service constraint on these contracts and uncertainty about the bank's ability to repay can lead to a ''bank run'' situation: All depositors rush to the bank at the same time to withdraw their money, trying to avoid being the last one in the waiting queue. This threat of bank runs creates an inherent instability for the bank's business (Diamond and Dybvig, 1983; Jacklin and Bhattacharya, 1988).

These characteristics highlight the major differences between banks and other firms: Banks, in contrast to other corporations, are financed by deposits. Their ongoing operating value would be lost to a large extent in case of bankruptcy; depositors can immediately call their claims and run whereas illiquid and information sensitive assets have to be liquidated by fire sales at significant costs (Diamond and Rajan (2000, 2001); Shrieves and Dahl (1992); the size of bankruptcy costs of banks was estimated in James (1991)). However, these features of a bank are ignored by most of the literature on capital structure and hedging decisions, which usually deals with nonfinancial firms.

In a recent contribution, Froot and Stein (1998) developed a framework to analyze a bank's optimal capital allocation, capital budgeting and risk management decisions. Their motivation for the bank to care about risk management stems from convex costs of external financing for a follow-up investment opportunity. This induces the bank's objective function to be concave (the authors call this internal risk aversion): The more difficult it is for the bank to raise external funds, the more risk averse it behaves. A publicly traded bank in an efficient and complete market does not reduce shareholder value by sacrificing return for a reduction in risk. Thus, risk reduction is always desirable for the risk-averse bank in the Froot and Stein (1998)-setting. Hence, the resulting optimal strategy is to hedge completely. However, the authors omit the equity's feature of limited liability and the corresponding agency problems between shareholders and debtholders. Furthermore, since in their model, there is no depository debt and thus no bank run possibility, potential effects of defaults on capital structure and risk management decisions are ignored.

In this paper, we model the hedging decision of a bank with the aforementioned characteristics. We assume the capital budgeting decision to be fixed. In a one-period-two-states-model, the bank has a given amount of depository debt. The deposit rate contains a discount due to deposit-related services. The present value of this discount constitutes the bank's franchise value. On the other hand, bank runs can force the bank to sell all of its assets at once, incurring significant liquidation costs. This creates an incentive for not having extraordinary high levels of depository debt. Further, we assume that the bank is restricted in its risk taking behavior by a regulator. We also incorporate limited liability for equity. We assume that the bank's

management acts in the shareholder's interest and maximizes the present value of the equity. It faces thus conflicting incentives for risk management: Regulatory restrictions and liquidation costs in case of bank runs limit the risk taking on one hand. On the other hand, limited liability creates incentives for risk taking. This setting allows us to identify situations in which well known results from the corporate finance literature are found: We show that for some banks, it is optimal to hedge completely as in Froot and Stein (1998). Other banks will take on as much risk as possible to augment shareholder value by expropriating wealth from depositors, a strategy known as asset substitution (Jensen and Meckling, 1976). For still other banks, the risk management decision is shown to be irrelevant as in Modigliani and Miller (1959).

The remainder of this paper is organized as follows. In Section 2, we present the model, discuss the bank's objective function and derive the optimal hedging strategy. In Section 3, we discuss the impact of forward counterparty restrictions on the hedging positions of the bank: Since depositors have absolute priority because of their possibility to withdraw at any time, the forward counterparty can face additional default risk. It may therefore limit its contract size with the bank. Section 4 concludes the analysis and gives an outlook on further research possibilities.

2. The general model

2.1. The market

Let a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ be given, where we define $\Omega := \{\mathcal{U}, \mathcal{D}\}$, $\mathcal{F} := \{\emptyset, \{\mathcal{U}\}, \{\mathcal{D}\}, \Omega\}$ and $\mathbb{P}(\mathcal{U}) = p$. The model has one period, between time $t = 1$ and $t = 2$ and $T \equiv \{1, 2\}$ denotes the set of time indices.

The market consists of two assets: A riskless asset has at time $t = 1$ a value normalized to 1, $B_1 = 1$, and $B_2 = B_1 R$ at time $t = 2$ where $R > 1$ is fixed and given; further, a risky asset with value $P_1 > 0$ at time $t = 1$ and a value $P_2(\omega)$ at time $t = 2$ where

$$P_2(\omega) = \begin{cases} P_u \equiv P_1 u, & \omega = \mathcal{U}, \\ P_d \equiv P_1 d, & \omega = \mathcal{D}, \end{cases}$$

where we assume that

$$u > R > d. \tag{1}$$

For hedging purposes, we further introduce a redundant forward contract on the risky asset: It is entered at time $t = 1$ at no cost and the buyer of the contract has to buy one unit of the risky asset at time $t = 2$ at the forward price RP_1. Hence, the value f_t of the forward contract is

$$f_1 = 0,$$

$$f_2(\omega) = \begin{cases} f_u \equiv P_u - RP_1, & \omega = \mathcal{U}, \\ f_d \equiv P_d - RP_1, & \omega = \mathcal{D}. \end{cases}$$

Since we have two assets with linearly independent payoffs and two states of the world, the market is complete. We define the unique risk neutral probability Q by $Q(\mathcal{U}) =: q$ such that $\mathbb{E}^Q[\frac{P_2}{B_2}] = P_1$, where $q = \frac{R-d}{u-d}$.

2.2. The bank

To derive the bank's objective function, we make the following two assumptions that deal with agency problems: We explicitly exclude agency problems between shareholders and bank managers as their decision-taking agents. However, since banks empirically have very high debt levels, we take asset substitution as an agency problem between shareholders and depositors into account. Therefore, the problem of choosing risk after the choice of the initial capital structure is especially pronounced (Leland, 1998):

Assumption 1. Management's compensation is structured to align the manager's interests with those of the shareholders. Therefore the firm's objective is to maximize the value of equity.

This objective is based on the completeness of the financial market. It is therefore possible to achieve any distribution of wealth across states. The Fisher separation theorem then states the following: All utility maximizing shareholders agree on the maximization of firm value as the appropriate objective function for the firm, notwithstanding the differing preferences and endowments (Eichberger and Harper, 1997, p. 150). However, as Jensen and Meckling (1976) pointed out, shareholders in levered firms can do better behaving strategically. They will prefer investment or hedging policies that maximize the value of only their claim, if they are not forced to a precommitment on the investment and hedging strategy.

Assumption 2. When setting its capital structure, the bank cannot precontract or precommit its hedging strategy. It will choose the hedging strategy ex post, after deposits have been raised.

At time $t = 1$, the bank has a loan portfolio, which has the same dynamics as the risky asset. Its value at $t = 1$ equals αP_1, we will thus say it has a prior position of $\alpha > 0$ units of the risky asset. [2] The bank has two sources of capital: Depository debt and equity where the latter has limited liability. The initial amount of depository debt D_1 is given. While it would also be interesting to analyze the bank's capital structure decision, we limit our analysis to the hedging policy, assuming that the bank has already set its target capital structure.

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[2] Through their monitoring activity, banks may be able to generate additional rents on the asset side as well. These proprietary assets are however often not tradeable. In this situation, the market is incomplete. This incompleteness creates problems for the determination of a unique objective function for the bank and we leave the analysis of the case with nontradeable proprietary assets for further research.

2.3. The deposits and the run-threat to equity

In most papers dealing with the capital structure of firms in general, the tax-advantage of debt is a main incentive for firms to carry debt. For banks, however, there is a more important motivation for carrying depository debt. Depository debt in banks can be regarded as a real production element (Bhattacharya and Thakor, 1993). Due to deposit-related services (liquidity provision, payment services), the deposit rate will be lower than the rate that fully reflects the risk. We assume that the bank gets a discount of $\tau > 0$ on the deposit rate, resulting in

$$D_2 = D_1 R_D,$$

where $R_D > 1$ is the deposit rate net of the discount that the bank receives. We call the net present value of these discounts from future periods the franchise value of the deposits, denoted by FV_t, $t \in T$:

$$FV_1 = \frac{\tau D_1}{B_2},$$

$$FV_2 = 0,$$

where $\frac{\tau D_1}{B_2} = \mathbb{E}^Q[\frac{\tau_1}{B_2}]$.

Because of its significant influence on the bank's equity payoff, we should highlight another important difference between bank deposits and traded debt: Asset-return shocks affect market prices of traded debt equally over all debt holders, whereas the nominal amount of deposits can be withdrawn at any time. However, as Diamond and Dybvig (1983) pointed out, the sequential service constraint on these fixed-commitment contracts along with sudden shocks in the liquidity needs of depositors can lead to a situation in which all depositors withdraw their money at the same time. This is because the amount received by a individual depositor solely depends on his relative position in the waiting queue. Such a bank run can happen as a "sunspot phenomenon", whenever there is a liquidity shock and even in the absence of risky bank assets. [3] When uncertain asset returns are introduced into the analysis, there is another reason why bank runs can occur: Whenever the value of the bank's assets is not sufficient to repay every depositor's full claim, all fully informed rational depositors would run to the bank at the same time and cause a so-called information based bank run (Jacklin and Bhattacharya, 1988).

Let us assume that there are n depositors with equal amounts of D_2/n of deposits. We denote by V_L the critical asset value below which there will be a bank run. Without liquidation costs, we find that $V_L = D_2$. Indeed, whenever the value V_2 of the bank's assets at time $t = 2$ exceeds the nominal deposits D_2, all depositors will receive their nominal claim. But as soon as the value V_2 of the bank's assets falls below the

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[3] The right to withdraw at any time is an essential prerequisite for the efficiency of the deposit contract. In accordance with Diamond and Rajan (2000), we therefore exclude the possibility of suspension of convertibility for the bank in our model; the bank cannot deny redemption of deposits as long as there are any assets left.
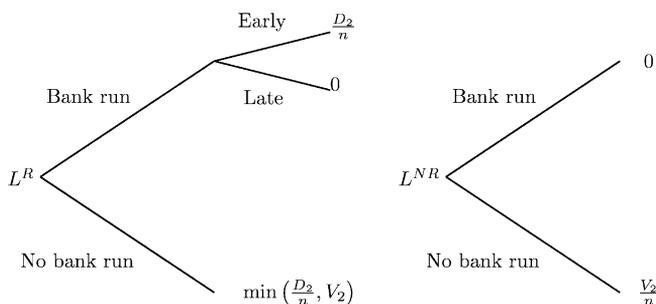
Fig. 1. Payoffs to a depositor in absence of liquidation costs.

nominal value D_2 of the deposits, not all depositors can withdraw their full nominal amount anymore. In the latter case, each depositor faces the problem of choosing between two compound lotteries: By running, he chooses the lottery L^R with payoffs depicted in Fig. 1. By not running, he chooses the lottery L^{NR} with payoffs also depicted in Fig. 1:

- When there is a bank run, the first V_2/D_2 percent of the depositors in the waiting queue receive their full nominal deposit D_2/n. Thus, if the individual depositor runs, the likelihood of arriving early at the queue (denoted 'early') is V_2/D_2. The payoff in this case is $\frac{D_2}{n}$. If he joins the queue in a later position (denoted 'late'), his payoff is 0. When there is no bank run, the individual depositor is the only to run and he receives his nominal deposit D_2/n or all of the assets remaining.
- When the individual does not run he either receives 0 if there is a bank run or $\frac{V_2}{n}$ if there is no bank run because in this case the value of the remaining assets is distributed equally among the depositors.

For $V_2 < D_2$, the payoffs of the run-strategy L^R are higher or equal to those of the no-run-strategy in all states of the world. Equivalently, the distribution of the run-strategy L^R first-order stochastically dominates the distribution of the no-run-strategy L^{NR}. Hence, every expected utility maximizing depositor with positive marginal utility will prefer the run-strategy L^R (see e.g. Mas-Colell et al., 1995). This leads to an equilibrium situation which is called information-based bank run.

In run situations, fire sales of assets necessary to pay out the depositors may create significant liquidation costs (indirect bankruptcy costs) on the other hand (Diamond and Rajan, 2001): Asset market prices can drastically decline if big blocks of assets have to be sold immediately. If the bank has to sell all of its assets at once during a run, we assume that there are liquidation costs of γV_2, $0 < \gamma < 1$. The fraction γ of firm value lost in case of bank runs creates a major incentive for the bank to hedge its risk: Averaging 30% of the bank's assets, these losses are substantial in bank failures as James (1991) found in his empirical work.

Since there is always a possibility of "sunspot"-bank runs due to unexpected liquidity shocks, the individual depositor is uncertain whether there will be a bank
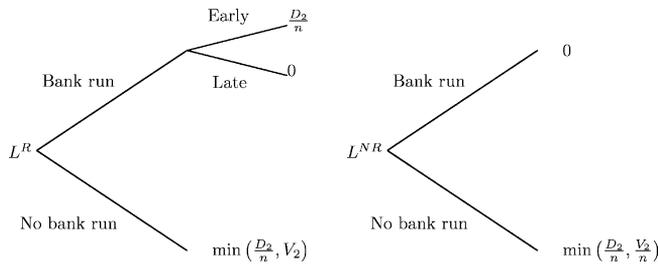
Fig. 2. Payoffs to a depositor in the presence of liquidation costs.

run at time $t = 2$ (Diamond and Dybvig, 1983). [4] Because of this uncertainty and the liquidation costs γV_2, V_L, the value of the assets below which an information based bank run will be triggered, shifts to

$$V_L = \frac{D_2}{1 - \gamma}; \tag{2}$$

for $D_2 < V_2 < \frac{D_2}{1-\gamma}$ we now have the payoffs given in Fig. 2. They resemble the payoffs shown in Fig. 1 without liquidation costs, but now total value of the assets is reduced to $(1 - \gamma)V_2$ instead of V_2 in situations of bank runs:

- When there is a bank run, the first $V_2(1 - \gamma)/D_2$ percent of the depositors in the waiting queue receive their full nominal deposit D_2/n. If the individual depositor runs, the likelihood of arriving 'early' is $V_2(1 - \gamma)/D_2$ and the payoff in that case is $\frac{D_2}{n}$. If he joins the queue in a later position (denoted 'late'), his payoff is 0. When there is no bank run, the individual depositor is the only one to run and he receives his nominal deposit D_2/n or all of the assets remaining.
- When the individual does not run, he receives 0 if there is a bank run. Otherwise he either receives his full nominal amount $\frac{D_2}{n}$ (if $D_2 \leqslant V_2 \leqslant V_L$), or a fraction $\frac{V_2}{n}$ of the remaining assets, which are distributed equally among the depositors.

Again, for $V_2 < \frac{D_2}{1-\gamma}$, the distribution of the run-strategy L^R first-order stochastically dominates the distribution of the no-run-strategy L^{NR}, causing an information-based bank run equilibrium.

Without this bank run-threat, the payoff function for the bank's equity at time $t = 2$ would be

$$S(V_2, D_2) \equiv \begin{cases} V_2 - D_2, & V_2 \geqslant D_2, \\ 0, & 0 \leqslant V_2 < D_2. \end{cases}$$

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[4] We assume that the bank can only raise deposits of the size $D_1 \leqslant (1 - \gamma)\alpha P_1$ such that bank runs at time $t = 1$ are excluded. This condition can equivalently be written as $\frac{D_1}{\alpha P_1} \leqslant (1 - \gamma)$ and interpreted in the following way: Banks can only raise deposits up to the point where the debt ratio equals the recovery rate in case of a run.
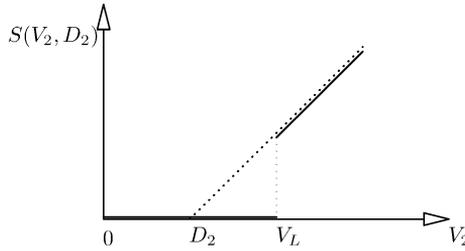
Fig. 3. Payoff function of equity.

This is the payoff of an ordinary call option on the firm value with strike D_2. However, in the presence of liquidation costs, a bank run will always take place if $V_2 < V_L$. Thus the residual payoff to shareholders drops to zero below V_L. Since $V_L > D_2$, the equity payoff changes to

$$S(V_2, D_2) \equiv \begin{cases} V_2 - D_2, & V_2 \geqslant V_L, \\ 0, & 0 \leqslant V_2 < V_L, \end{cases} \tag{3}$$

as shown in Fig. 3.

2.4. The optimization problem

At time $t = 1$, the bank chooses a hedging position consisting of h units of the forward contract on the risky asset. As a function of the chosen hedging position h, the value of the bank's assets at time $t = 1$ hence is

$$V_1(h) = \alpha P_1 + h f_1 + FV_1, \quad h \in \mathbb{R}, \tag{4}$$

and the value of the bank's assets in state $\mathscr{U}$ and $\mathscr{D}$ respectively at time $t = 2$ for a given hedging position h is denoted by

$$V_u(h) \equiv \alpha P_u + h f_u, \quad h \in \mathbb{R}, \tag{5}$$

$$V_d(h) \equiv \alpha P_d + h f_d, \quad h \in \mathbb{R}. \tag{6}$$

To study the impact of regulatory or other restrictions on the risk management, we introduce lower and upper bounds [5] on the hedging position h,

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[5] Liquidation costs γV_2 are expressed as a fraction of the final firm value V_2. Thus, by introducing the following (merely technical) restriction on the bank's hedging decision, admitting only hedging strategies for which the firm value is always positive, we guarantee nonnegative liquidation costs. This amounts to the restriction $h \in [Z^u, Z^d]$ where $Z^u \equiv -\alpha \frac{u}{u-R}$ and $Z^d \equiv -\alpha \frac{d}{d-R}$. Indeed, these constants follow from solving the inequalities $V_u(h) \geqslant 0$ and $V_d(h) \geqslant 0$ using the definitions (5) and (6) of $V_u(h)$ and $V_d(h)$. $V^u(h) \geqslant 0$ holds for $h \in [Z^u, \infty)$ and $V^d(h) \geqslant 0$ holds for $h \in (-\infty, Z^d]$, thus nonnegative firm value is the outcome for hedging strategies in $(-\infty, Z^d] \cap [Z^u, \infty)$. It follows from the definition of Z^u that $Z^u < -\alpha$ thus $-\alpha$ as the lower bound of the set of feasible hedging positions guarantees nonnegative firm value in state $\mathscr{U}$. Throughout the following we assume that the upper bound is more restrictive than Z^d, $a_1 < Z^d$.

$$-\alpha \leqslant h \leqslant a_1. \tag{7}$$

The lower bound $-\alpha$ is equivalent to a no net-shortsales-constraint. [6]

The payoff S to shareholders at liquidation at time $t = 2$ is a function of firm value and deposits, $S(V_2, D_2)$. Since the financial market is arbitrage-free and complete, the present value of equity at time $t = 1$ for a given future value V_2 of the assets equals

$$\mathbb{E}^Q\left[\frac{S(V_2, D_2)}{B_2}\right].$$

The bank's management's goal is to maximize the present value of equity at time $t = 1$, by choosing a hedging position h. Let

$$I(h) \equiv \mathbb{E}^Q\left[\frac{S(V_2(h), D_2)}{B_2}\right] \tag{8}$$

denote the objective function. $I(h)$ is the value of equity at time $t = 1$ as a function of the hedging portfolio h. Then, the bank's optimization problem at time $t = 1$ is

$$\max_{-\alpha \leqslant h \leqslant a_1} I(h) = \max_{-\alpha \leqslant h \leqslant a_1} \mathbb{E}^Q\left[\frac{S(V_2(h), D_2)}{B_2}\right]$$
$$= \max_{-\alpha \leqslant h \leqslant a_1} \mathbb{E}^Q\left[\frac{S(\alpha P_2 + h f_2, D_2)}{B_2}\right]. \tag{9}$$

2.5. Optimal hedging strategy

The present value of equity as a function of the hedging position h, $I(h)$, has the following form, depending on whether for a given hedging position h there will be a positive payoff to shareholders in both states $\mathcal{U}$ and $\mathcal{D}$ or only in state $\mathcal{U}$:

1. For hedging positions h such that the assets' total value exceeds the bank run trigger V_L in both states $\mathcal{U}$ and $\mathcal{D}$, $V_u(h) > V_L$ and $V_d(h) > V_L$, amounting to a positive payoff to shareholders in both states (we call these hedging portfolios portfolios of type 1), the objective function is

$$I(h) = \frac{q}{B_2}[\alpha P_u + h(P_u - RP_1) - D_2] + \frac{1-q}{B_2}[\alpha P_d + h(P_d - RP_1) - D_2].$$

2. For hedging positions h such that the assets' total value exceeds only in state $\mathcal{U}$ the bank run trigger V_L, $V_u(h) > V_L$, but is smaller than V_L in state $\mathcal{D}$, $V_d(h) < V_L$ (that is, there is a bank run in state $\mathcal{D}$ and shareholders receive only in state $\mathcal{U}$ a positive payoff; we call these hedging positions portfolios of type 2), the objective function is

$$I(h) = \frac{q}{B_2}[\alpha P_u + h(P_u - RP_1) - D_2].$$

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[6] The net position in the risky asset is restricted to be nonnegative and thus, the cases where $V_u(h) < V_d(h)$ can be omitted.

Thus, in order to solve the optimization problem, we need to find conditions which guarantee the existence of hedging positions h of either type 1 or type 2 for a given market and bank structure.

Lemma 1. *The payoff to shareholders is positive in state $\mathscr{U}$ for hedging positions h*

$$h \geqslant \frac{V_L - \alpha P_u}{P_u - RP_1} =: K^u. \tag{10}$$

The payoff to shareholders is positive in state $\mathscr{D}$ for hedging positions h

$$h \leqslant \frac{V_L - \alpha P_d}{P_d - RP_1} =: K^d. \tag{11}$$

K^u is the minimal hedging position for which shareholders receive a positive payoff in state $\mathscr{U}$. K^d is the maximal hedging position for which shareholders receive a positive payoff in state $\mathscr{D}$. Hence, the portfolios of type 1 are in the set $[K^u, K^d]$. Therefore, the relationship among the terms K^u and K^d will determine whether this set is empty and whether there are hedging positions of type 1 or only of type 2.

Corollary 1. *Hedging positions h of type* 1 *exist if*

$$V_L \leqslant \alpha P_1 R =: \overline{V}. \tag{12}$$

$\overline{V}$ is the value of the assets of the 'fully hedged bank' at time $t = 2$, i.e. the value attained if the bank sells forward its whole position α in the risky asset and the future value becomes certain, $V_u(-\alpha) = V_d(-\alpha) = \alpha P_1 R = \overline{V}$. Thus, if the bank run trigger V_L is smaller or equal to the forward price $\overline{V}$ of the bank's prior position, there exist hedging positions for which shareholders receive a positive payoff in both states $\mathscr{U}$ and $\mathscr{D}$. It is obvious that the fully hedged position $-\alpha$ would be such a position. And if the payoff to shareholders with this forward position is strictly positive, there will be other forward positions close to $-\alpha$ which also yield a positive payoff to shareholders. Otherwise, if the forward price $\overline{V}$ of the bank's prior position is smaller than the bank run trigger V_L then there are only hedging positions of type 2. Shareholders then receive a positive payoff only in state $\mathscr{U}$ and a zero payoff in state $\mathscr{D}$, $V_u(h) \geqslant V_L$ and $V_d(h) < V_L$.

The shape of the objective function is further clarified by the following

Lemma 2. *If $V_L \geqslant \overline{V}$, then the inequality*

$$K^d \leqslant -\alpha \leqslant K^u \tag{13}$$

holds. Otherwise, if $V_L \leqslant \overline{V}$, then the inequality

$$K^u \leqslant -\alpha \leqslant K^d \tag{14}$$

holds.

The intuition for (13) and (14) is as follows:

- If $V_L > \overline{V}$, then the fully hedged position $-\alpha$, leading to a value of $\overline{V}$ in both states of the world, results in a payoff of zero to shareholders, since the firm value is in this case below the bank run trigger V_L. Thus, the minimal (maximal) hedging position at which shareholders receive a positive payoff in state $\mathcal{U}$ ($\mathcal{D}$), i.e. K^u (K^d), would be higher (smaller) than $-\alpha$. This corresponds to (13).
- The converse holds if $\overline{V} \geqslant V_L$. Then the fully hedged position $-\alpha$ yields a positive payoff to shareholders. Then the minimal (maximal) hedging position at which shareholders receive a positive payoff in state $\mathcal{U}$ ($\mathcal{D}$), i.e. K^u (K^d), would be smaller (higher) than $-\alpha$. This corresponds to (14).

Fig. 4 displays the objective function $I(h)$ with the no net short sales restriction in the three cases where $V_L > \overline{V}$ (Fig. 4(a)), $V_L < \overline{V}$ (Fig. 4(b)) and $V_L = \overline{V}$ (Fig. 4(c)). The bold line is the feasible part of the objective function.

(a) $V_L > \bar{V}$ ($K^d < K^u$)

(b) $V_L < \bar{V}$ ($K^u < K^d$)
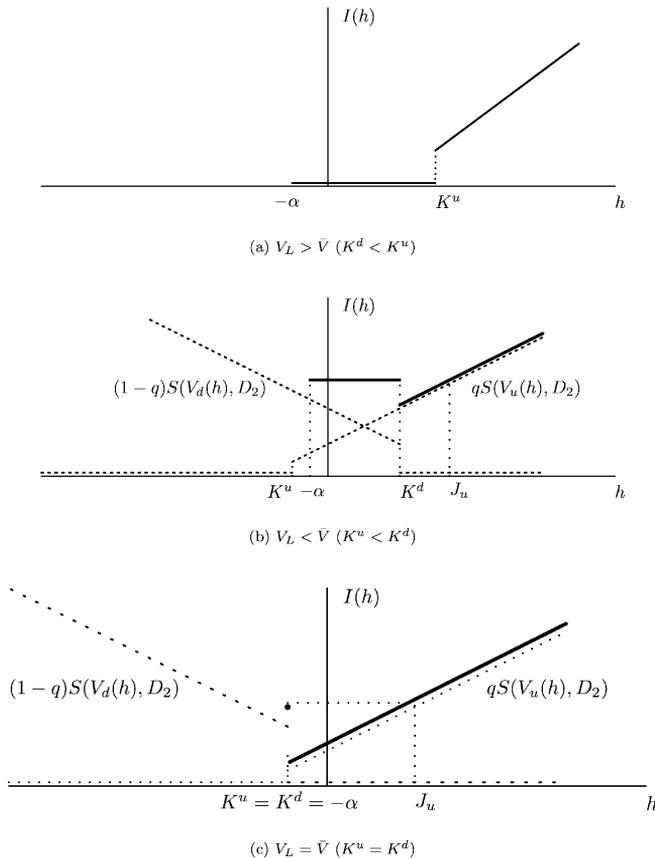
(c) $V_L = \bar{V}$ ($K^u = K^d$)

Fig. 4. Three types of objective functions with no net short sales restrictions.

Proposition 1

1. *If the forward price of the bank's prior position is less than the bank run trigger, $\alpha P_1 R < V_L$, and if there is a positive payoff to shareholders in state $\mathcal{U}$ at the maximal admissible hedging position, $a_1 \geqslant K^u$, then the optimal hedging position is*

   $$h^* = a_1.$$

2. *Otherwise, if $\alpha P_1 R > V_L$, we find the following optimal hedging strategies*:
   (a) *If $a_1 \geqslant K^d$ and $a_1 > J_u$, then $h^* = a_1$, where J_u is defined by (15) below.*
   (b) *If $J_u > a_1 > K^d$, then $-\alpha \leqslant h^* \leqslant K^d$.*
   (c) *If $a_1 < K^d$, then $-\alpha \leqslant h^* \leqslant a_1$.*
3. *If $\alpha P_1 R = V_L$, we find the following optimal hedging strategies*:
   (a) *If $a_1 < J_u$, then $h^* = -\alpha$.*
   (b) *If $a_1 > J_u$, then $h^* = a_1$.*

J_u can be characterized as follows (see Fig. 4(b)): The position $h = J_u$ belongs to the set of hedging positions for which the objective function is increasing; further, J_u is the position for which the value function equals the value that is attained on the set where the objective function is constant,

$$J_u \equiv \frac{\alpha P_1 - \dfrac{D_2}{R} - q\dfrac{(\alpha P_u - D_2)}{R}}{q\dfrac{P_1(u - R)}{R}}. \tag{15}$$

The three optimal hedging decisions in Proposition 1 have the following economic interpretation:

- $h = a_1$ is the strategy of maximal speculation.
- $h = -\alpha$ is the case of complete hedging where the bank sells forward its whole initial position.
- In the case where $K^u < K^d$, the bank is indifferent between the hedging strategies in the range $K^u \leqslant h \leqslant K^d$.

Part 1 of Proposition 1 covers the case in which the payoff to shareholders would be zero if the bank hedged completely. It is the case in which the forward price of the prior position is less than the bank run trigger, $\overline{V} < V_L$. If a positive payoff to shareholders in state $\mathcal{U}$ is attainable by taking on more risk, $a_1 \geqslant K^u$, we have a "gamble for resurrection"-situation: It is always optimal to take as much risk as possible, $h^* = a_1$. The condition $V_L > \overline{V}$ can equivalently be written as

$$D_1/\alpha P_1 > (R/R_D)(1 - \gamma), \tag{16}$$

which says that the initial debt ratio is higher than the recovery rate (in case of a run) multiplied by the spread between the deposit rate and the riskless interest rate. Hence, for banks with high initial debt ratio and/or high liquidation costs, it is always optimal to gamble for resurrection.

Parts 2a to 2c of Proposition 1 cover the cases in which shareholders would still receive a positive payoff if the bank hedged completely, $V_L < \overline{V}$, resp. $D_1/\alpha P_1 < (R/R_D)(1 - \gamma)$.

- In 2a, the maximal admissible hedging position a_1 yields a higher expected payoff than the 'fully hedged' position $-\alpha$. Since shareholders could lock in a sure positive payoff by hedging completely, this is not a "gamble for resurrection", although the optimal hedging strategy is the same. Due to equity's nonlinear payoff, they can expropriate wealth from depositors by taking on more risk: The increase of the payoff in state $\mathscr{U}$ overcompensates the liquidation costs in state $\mathscr{D}$. This strategy is known as asset substitution (Jensen and Meckling, 1976). Hence, in the current model, banks in this situation have loose regulatory or other restrictions (a large a_1). They can take that much risk that the bank run threat does not have a disciplinary effect anymore.
- In 2b, the risk management restriction is so constraining, that at the maximal admissible position a_1, the gain in expected return does not outweigh the expected liquidation costs of this portfolio. The expected payoff to shareholders for this hedging position is smaller than for the 'fully hedged' position $-\alpha$. However, there is no unique optimal hedging strategy: Shareholders are indifferent with respect to the hedging strategies in the whole range between $-\alpha$ and K^d. If the initial debt ratio $\frac{D_1}{\alpha P_1}$ is higher than $\frac{d}{R_D}(1 - \gamma)$, then $K^d < 0$ and the optimal hedging strategy is risk reducing, $h^* < 0$. Risk reducing banks in this case are those with a high initial debt ratio, high asset volatility and/or high liquidation costs.
- In 2c, the maximal admissible hedging position a_1 belongs to the portfolios for which shareholders receive a positive payment in both states $\mathscr{U}$ and $\mathscr{D}$. The expected payoff is the same as the one of the 'fully hedged' position $-\alpha$. In this case, the Modigliani–Miller-result of hedging-irrelevance also holds ex post, after the determination of the capital structure: Shareholders are indifferent with respect to all admissible hedging strategies. Banks in this case are, however, forced towards a safe behavior: The risk management restrictions prevent asset substitution since they guarantee that the value of banks' assets can never fall below the bank run trigger.

In part 3a, the 'fully hedged' position $h^* = -\alpha$ is optimal. Any risk taken by the bank induces liquidation costs. But the expected return cannot be increased sufficiently such that the shareholders would receive a higher expected payment at least in one state since $J_u > a_1 > K^d$. Banks in this situation do not have any risk tolerance. They cannot improve the shareholders' position by asset substitution. In our model, only for this special situation, the Froot and Stein (1998)-result of complete hedging is derived as the unique optimal hedging strategy.

In part 3b, the regulatory constraint a_1 is loose enough to allow the bank to take on enough risk such that the expected return again outweighs the expected liquidation costs.

Overall, for a regulatory restricted bank financed with deposits that is subject to liquidation costs in the event of bank runs, the common interpretation of equity as a

call option does not necessarily apply: Equity value is not always increased by an increase in asset-risk. Further, higher liquidation costs lead to an increase of the bank run trigger. This creates larger downside risk for shareholders that cannot always be outweighed by a higher expected return, because regulatory restrictions place an upper limit on risk taking.

On the other hand, depending on how much risk taking regulatory or other restrictions allow, hedging completely as in Froot and Stein (1998) is almost never the unique optimal hedging strategy: Over a wide range, all hedging positions can be equally optimal. Risk shifting to depositors is optimal as long as the higher expected return outweighs the possible downside loss. If risk management restrictions are set to prevent asset substitution, the value of the bank's assets cannot fall below the bankrun trigger. The result then coincides with the Modigliani–Miller-result of hedging irrelevance.

3. Impact of counter party risk constraints

We extend the analysis of the previous section by introducing counter party restrictions on the attainable forward contract size used for hedging. The forward price RP_1 is set such that expected profit from the forward contract is zero under the risk neutral probability measure Q. Yet, if the bank can default on the forward contract, the counter party will demand a higher forward price to get compensated for the additional risk. If we leave the forward price fixed, the bank will not be able to enter every desired forward position any more. The counter party restricts the hedging decision by offering only forward contracts for which the probability of default does not exceed some threshold. In the current binomial setting, statements on probabilities correspond to conditions on states $\mathscr{U}$ and $\mathscr{D}$:

- Zero probability of default is equivalent to no default in both states $\mathscr{U}$ and $\mathscr{D}$.
- If the probability of default can be positive, then the bank is not allowed to default either in state $\mathscr{U}$ or in state $\mathscr{D}$.

Proposition 2. *The bank will not default on the forward contract in state $\mathscr{U}$ for contracts of size $h \geqslant K^u$. Further, the bank will not default on the forward contract in state $\mathscr{D}$ for contracts of size $h \leqslant K^d$. Under the requirement that the bank should not default in any state of the world on its obligations from the forward contract, it will not be able to enter a forward contract unless $K^u \leqslant K^d$. It will only be offered contracts h such that $K^u \leqslant h \leqslant K^d$.*

The question when the bank is offered both long and short or only long or only short positions is answered by the following

Lemma 3. *If the bank is not allowed to default in state $\mathscr{U}$ (state $\mathscr{D}$), it will be offered short (long) positions if and only if $V_L < \alpha P_1 u$ ($V_L < \alpha P_1 d$).*

Hence, the restriction not to default in state $\mathscr{D}$ may prevent the bank to enter long positions, namely if $D_1/\alpha P_1 > (1 - \gamma)(d/R_D)$. These banks either have a high debt ratio, high liquidation costs and/or a high asset-volatility. They would face a bank run in state $\mathscr{D}$ without hedging and the costs would be borne by the counter party. On the other hand, the restriction not to default in state $\mathscr{U}$ may prevent the bank to enter short positions (if $D_1/\alpha P_1 > (1 - \gamma)(u/R_D)$). The debt ratio, the liquidation costs and/or the asset-volatility of these banks is that high that they would face a bank run already in the 'good' state of the world $\mathscr{U}$ and the counter party enforces the asset substitution in this case. The most important type of restriction [7] is the one which does not allow default in any state. In the case where $V_L > \overline{V}$, [8] the bank cannot enter a forward contract. With its combination of deposits, initial position and liquidation costs, it will not be offered forward contracts due to default risk. Thus, the gamble for resurrection is not possible any more. When $V_L \leqslant \overline{V}$, the bank is prevented from taking on any risk which would trigger a bank-run. It can only enter positions in the forward in the range $-\alpha \leqslant h \leqslant K^d$. Thus, the bank will always have the possibility to reduce risk by entering short positions. In the subcase where $V_L > \alpha P_1 d$, it will not be able to obtain long positions (Lemma 3). That is, when the bank's prior position is sufficient to prevent a bank run only in state $\mathscr{U}$, but not in state $\mathscr{D}$, the bank will only be offered contracts that reduce the risk sufficiently to ensure that there will be no bank run in state $\mathscr{D}$. Without hedging, the bank would face a bank run in state $\mathscr{D}$. But with the positive cash flow $-(P_d - RP_1)$ from the short position in the forward contract in state $\mathscr{D}$, the bank's assets are sufficient to prevent a run in state $\mathscr{D}$. The following Lemma tells when the bank will choose to hedge.

Lemma 4. *In the case where $V_L \leqslant \overline{V}$ and $\alpha P_1 d < V_L$, the bank will choose to hedge if $D_2 \leqslant \alpha P_1 d < V_L$; if $\alpha P_1 d < D_2$, then it is optimal for the bank not to hedge.*

The reason for this hedging-strategy is the following: By hedging when $D_2 \leqslant \alpha P_1 d < V_L$, the bank can preserve asset value in the down state $\mathscr{D}$, that otherwise would be completely lost for the shareholders as liquidation costs. On the other hand, if $\alpha P_1 d < D_2$, all the remaining asset value up to D_2 goes to the depositors anyway. If the bank hedges, it thus sacrifices some payoff to shareholders in state $\mathscr{U}$ in exchange

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[7] The constraint that the bank should not default in state $\mathscr{U}$ but is allowed to default in state $\mathscr{D}$ is only meaningful if the risk neutral probability of state $\mathscr{D}$ is very low. The forward contract price is then approximately not affected by the additional default risk. The following results then apply: If $V_L > \overline{V}$, the bank can obtain only positions $h \geqslant K^u$, since it would default in state $\mathscr{U}$ on all other positions. Therefore, the bank can still follow a strategy of asset substitution by holding long positions in the forward contract. If even $V_L \geqslant \alpha P_1 u$, that is, if the bank faced a bank run without hedging in state $\mathscr{U}$, it would only be offered long positions to hedge and thus be forced to 'gamble for resurrection': For sufficiently large hedging positions, the value of the bank's asset is above the bank run trigger in state $\mathscr{U}$ (whereas in state $\mathscr{D}$, the bank will default on its obligation from the forward contract). In the case where $V_L \leqslant \overline{V}$, the constraint that the bank is not allowed to default in state $\mathscr{U}$ is not binding: It will be offered any contracts of size $h \geqslant K^u$ but the lower bound on its hedging position h already is $-\alpha$ where $-\alpha \leqslant K^u$ ((14) in Lemma 2).

[8] $V_L > \overline{V}$ implies $K^d < K^u$, and if follows from Proposition 2 that the bank will not be offered forward contracts.

for securing payoffs for depositors in state $\mathscr{D}$. The bank can do better for the shareholders by not hedging at all, that is, by keeping the higher expected return of the unhedged position while letting the depositors bear the downside loss in state $\mathscr{D}$.

Overall, the introduction of counterparty-restrictions mitigates risk taking incentives for a bank, since it is not possible to gamble for resurrection anymore.

4. Conclusion

We have presented a one-period model in which we analyze the bank's risk management decision. The bank is regulatory restricted, financed by deposits and is subject to liquidation costs in the event of a bank run.

We find that the common interpretation of equity as an ordinary call option does not apply: Equity value is not always increased by increasing the asset's volatility, since this also raises the likelihood of a bank run. Whenever the expected costs of such a run for shareholders cannot be outweighed by an increase of the expected return (because regulatory restrictions limit the maximum achievable risk), it is not optimal to take as much risk as possible. In these cases, safe banks with low debt ratios and asset volatility can still augment their risk exposure to the point where downside loss comes into play. However, for banks with a high debt ratio and a high asset volatility, risk reduction is the optimal strategy.

This deterrence of asset substitution however vanishes in the absence of regulatory constraints or with a complete deposit insurance (Calomiris and Kahn, 1991): Without the possible downside loss, the equity payoff would be that of an ordinary call option and it would always be optimal for the bank to take as much risk as possible. Also, without regulatory restrictions, the possible downside loss could always be outweighed by higher expected return through higher risk-exposures.

On the other hand, depending on how much risk taking regulatory or other restrictions allow, it may not be optimal for the bank to hedge completely as in Froot and Stein (1998): Because equity features limited liability, risk shifting to depositors is still preferred as long as the higher expected return outweighs the possible downside loss. The less restrictive regulatory restrictions are, the more relevant becomes this strategy of asset substitution. Without any restrictions of regulators or counter parties, asset substitution would always be the optimal strategy.

Further, there is one constellation for which the hedging decision is shown to be irrelevant, which coincides with the result of the Modigliani–Miller-theorem. This, however, is only a special situation, where the risk management restrictions, the size of the liquidation costs in case of a bank run and the initial debt ratio are all set such that risk shifting to depositors is impossible and no bank run takes place.

Among the open questions remains the analysis of the hedging decision in a multiperiod setting. Bauer and Ryser (2002) have looked at the effect that the bank's franchise value of deposits then has. It gives an incentive to reduce risk taking since the whole stream of future income from deposit services would be lost in a run situation. Furthermore, it would be interesting to analyze the hedging decision in the presence of a nontradeable proprietary bank asset that generates an extra rent as

in Diamond and Rajan (2000). The market completeness breaks down in this case and the determination of a unique objective function for the bank is not trivial anymore.

5. Proofs

Proof of Lemma 1. Using the definition (5) of $V_u(h)$ we find that for a given h

$$V_u(h) > V_L \iff h > \frac{V_L - \alpha P_u}{P_u - RP_1} = K^u;$$

similarly for a given h

$$V_d(h) > V_L \iff h < \frac{V_L - \alpha P_d}{P_d - RP_1} = K^d. \qquad \square$$

Proof of Corollary 1. From Lemma 1 we know that $V_u(h) > V_L$ for $h \in [K^u, \infty)$ and $V_d(h) > V_L$ for $h \in (-\infty, K^d]$. Hence, hedging positions h of type 1 (that is, h for which both $V_u(h) > V_L$ and $V_d(h) > V_L$) are $h \in [K^u, K^d]$; this interval is not empty if

$$K^u \leqslant K^d. \tag{17}$$

Using the definitions (10) and (11) of K^u and K^d this can be written equivalently as $\frac{V_L - \alpha P_u}{P_u - RP_1} \leqslant \frac{V_L - \alpha P_d}{P_d - RP_1}$. Solving for V_L yields $V_L \leqslant \alpha P_1 R = \overline{V}$. $\square$

Proof of Lemma 2. Consider first the case $V_L > \overline{V}$. From the definition (12) follows $V_L \geqslant \overline{V} = \alpha P_1 R$. Subtracting αP_u yields $V_L - \alpha P_u \geqslant -\alpha(P_u - P_1 R)$, dividing by $(P_u - P_1 R)$ yields $K^u \geqslant -\alpha$. The inequalities for K^d and for the case where $V_L \leqslant \overline{V}$ follow in the same way. $\square$

The following lemma will be useful to prove Proposition 1.

Lemma 5. *The sets of candidates for the optimal hedging strategy h^* are $\{a_1\}$, $\{-\alpha\}$ and $(-\infty, K^d] \cap [K^u, \infty)$. The values of the objective function evaluated at these candidate points are*

$$I(a_1) = \begin{cases} \frac{q}{B_2}[\alpha P_u + a_1(P_u - RP_1) - D_2], & K^d \leqslant K^u \leqslant a_1 \quad \text{or} \quad K^u \leqslant K^d < a_1, \\ \alpha P_1 - \frac{D_2}{B_2}, & K^u \leqslant a_1 \leqslant K^d, \\ 0, & K^d < a_1 < K^u, \end{cases} \tag{18}$$

$$I(-\alpha) = \begin{cases} \alpha P_1 - \frac{D_2}{B_2}, & K^u < K^d, \\ 0, & K^d < K^u, \end{cases} \tag{19}$$

$$I(h) = \begin{cases} \alpha P_1 - \frac{D_2}{B_2}, & K^u \leqslant h \leqslant K^d, \\ 0, & K^d < h < K^u. \end{cases} \tag{20}$$

Proof of Lemma 5. For convenient notation, we write the constraints (7) $-\alpha \leqslant h \leqslant a_1$ in the form

$$Ah \leqslant a, \tag{21}$$

where $A \equiv (A_1 \quad A_2)' = (1 \quad -1)'$ and $a \equiv (a_1 \quad -\alpha)'$.

We consider first the case $V_L \leqslant \overline{V} = \alpha P_1 R$. From $V_L < \overline{V} = \alpha P_1 R$ and (17) follows that $K^u \leqslant K^d$. Hence, for any $h \in [K^u, K^d]$ holds that $V_u(h) \geqslant V_L$ and $V_d(h) \geqslant V_L$. On this interval, the Lagrangian is as follows:

$$\mathscr{L}(h, \mu_1, \mu_2) = \frac{q}{B_2}[\alpha P_u + h(P_u - RP_1) - D_2] + \frac{1-q}{B_2}[\alpha P_d + h(P_d - RP_1) - D_2]$$

$$+ \sum_{k=1}^{2} \mu_k(A_k h - a_k)$$

$$= (\alpha + h)P_1 - \frac{hP_1}{B_1} - \frac{D_2}{B_2} + \sum_{k=1}^{2} \mu_k(A_k h - a_k),$$

yielding the first-order conditions

$$\frac{\partial \mathscr{L}}{\partial h} = P_1 - \frac{P_1}{B_1} + \sum_{k=1}^{2} \mu_k A_k = 0, \tag{22}$$

$$\frac{\partial \mathscr{L}}{\partial \mu_k} = A_k h - a_k = 0, \quad k = 1, 2.$$

We have the following candidate points for an optimum:

1. $\mu_1 = \mu_2 = 0$: (22) reduces to the condition $P_1 - \frac{P_1}{B_1} = 0$ where the last equality is due to $B_1 = 1$. Thus, in this case any h such that $V_u(h) \geqslant V_L$ and $V_d(h) \geqslant V_L$ is optimal, that is, the set of candidates is $[K^u, K^d]$. From $B_1 = 1$ follows $I(h) = \alpha P_1 - \frac{D_2}{B_2}$, $K^u \leqslant h \leqslant K^d$.

2. If $\mu_1 \neq 0$, $\mu_2 = 0$ then $h^* = a_1$. Then, if $K^u < K^d < a_1$ $I(a_1) = \frac{q}{B_2}[\alpha P_u + a_1(P_u - RP_1) - D_2]$ since the payoff to shareholders is zero in state $\mathscr{D}$ for $h = a_1$ due to the fact that $K^d < a_1$. If $K^u \leqslant a_1 \leqslant K^d$

$$I(a_1) = \frac{q}{B_2}[\alpha P_u + a_1(P_u - RP_1) - D_2] + \frac{1-q}{B_2}[\alpha P_d + a_1(P_d - RP_1) - D_2]$$

$$= \alpha P_1 - \frac{D_2}{B_2}.$$

3. If $\mu_1 = 0$, $\mu_2 \neq 0$ then $h^* = -\alpha$. Then, since $K^u \leqslant -\alpha \leqslant K^d$,

$$I(-\alpha) = \frac{q}{B_2}[\alpha P_u - \alpha(P_u - RP_1) - D_2] + \frac{1-q}{B_2}[\alpha P_d - \alpha(P_d - RP_1) - D_2]$$

$$= \alpha P_1 - \frac{D_2}{B_2}.$$

Consider now the case $V_L > \alpha P_1 R$ when there are only h such that $V_u(h) > V_L$ and $V_d(h) < V_L$ holds for fixed h. The Lagrangian is then

$$\mathscr{L}(h, \mu_1, \mu_2) = \frac{q}{B_2}[\alpha P_u + h(P_u - RP_1) - D_2] + \sum_{k=1}^{2} \mu_k(A_k h - a_k),$$

yielding the first-order conditions

$$\frac{\partial \mathscr{L}}{\partial h} = \frac{q}{B_2}[P_u - RP_1] + \sum_{k=1}^{2} \mu_k A_k = 0. \tag{23}$$

We have the following candidate points for an optimum:

1. $\mu_1 = \mu_2 = 0$ could hold only if $\frac{P_u}{P_1} = R$ which was excluded in (1).
2. $\mu_1 \neq 0$, $\mu_2 = 0$ and $h^* = a_1$. Then, if $K^d < K^u \leqslant a_1$, $I(a_1) = \frac{q}{B_2}[\alpha P_u + a_1(P_u - RP_1) - D_2]$ since the payoff to shareholders is zero in state $\mathscr{D}$ due to the fact that $K^d < a_1$. If $K^d < a_1 < K^u$ we have $I(a_1) = 0$ since the payoff to equity holders is both zero in state $\mathscr{D}$ (from $K^d < a_1$) and in state $\mathscr{U}$ (from $a_1 < K^u$).
3. $\mu_1 = 0$, $\mu_2 \neq 0$ and $h^* = -\alpha$. Then, it follows that $I(-\alpha) = 0$ since it follows from (13) that $K^d < -\alpha$ and hence at the position $-\alpha$ shareholders receive a zero payoff in state $\mathscr{D}$.

As for the last equality, it is obvious that for $h > K^d$, the payoff to shareholders is zero in state $\mathscr{D}$ and for $h < K^u$, the payoff to shareholders is zero in state $\mathscr{U}$, hence for $K^d < h < K^u$ follows $I(h) = 0$. $\square$

Proof of Proposition 1. We consider first part 1 of the proposition, that is, the case where $V_L > \alpha P_1 R$ and $a_1 \geqslant K^u$. As this is the case when $K^d < K^u$, it follows from (20) that the objective function equals zero for all $h \in (K^d, K^u)$ including $-\alpha$ (since, from Lemma 2, $K^d < -\alpha < K^u$). Definition (10) of K^u yields for $a_1 \geqslant K^u$ $a_1 \geqslant \frac{V_L - \alpha P_u}{P_u - RP_1}$ or $\alpha P_u + a_1(P_u - RP_1) \geqslant V_L$. From (18), $I(a_1) = \frac{q}{B_2}[\alpha P_u + a_1(P_u - RP_1) - D_2] > 0$ since $V_L > D_2$, hence, a_1 is the optimum.

We now turn to parts 2 and 3 of the proposition; both cases are covered by the inequality $V_L \leqslant \alpha P_1 R$ or equivalently $K^u \leqslant K^d$, 2 being the case of strict inequality and 3 the case of equality. Consider first 2a and 3b respectively. From (20) follows that for $h \in [K^u, K^d]$ (and hence also for $h = -\alpha$) $I(h) = \alpha P_1 - \frac{D_2}{B_2}$. From $J_u < a_1$ follows $\alpha P_1 - \frac{D_2}{B_2} < \frac{q}{B_2}[\alpha P_u + a_1(P_u - RP_1) - D_2] = I(a_1)$, hence $h^* = a_1$.

Similarly follows for 2b and 3a when $J_u > a_1 > K^d$ that $\alpha P_1 - \frac{D_2}{B_2} > \frac{q}{B_2}[\alpha P_u + a_1(P_u - RP_1) - D_2] = I(a_1)$, hence $h^* \in [-\alpha, K^d]$.

In part 2c, feasible portfolios h are $h \in [-\alpha, a_1] \subset (K^u, K^d)$. Hence, from (20), all feasible portfolios have the same value of the objective function, $I(h) = \alpha P_1 - \frac{D_2}{B_2}$, $K^u < h < K^d$, $h \in (K^u, K^d)$. $\square$

Proof of Proposition 2. If the bank should not default in state $\mathscr{U}$ on the forward contract, then the bank's assets net of the value of debt in state $\mathscr{U}$ must be positive, $\alpha P_u + h f_u - D_2 \geqslant 0$. Solving for h yields the required inequality $h \geqslant K^u$. In the case where the bank should not default in state $\mathscr{D}$ on the forward contract, the bank's assets net of the value of debt in state $\mathscr{D}$ must be positive, $\alpha P_d + h f_d - D_2 \geqslant 0$. Solving for h yields again the required inequality $h \leqslant K^d$. If the counter party of the forward contract requires that the bank does not default in any state, then h needs to be in the intersection of the intervals $[K^u, \infty)$ and $(-\infty, K^d]$. If $K^d < K^u$, then this intersection is empty, hence the bank will not be able to enter a forward contract. If $K^d > K^u$, then the intersection is exactly $[K^u, K^d]$. $\quad\square$

Proof of Lemma 3. By (1) $P_u = P_1 u > RP_1$ holds and thus $P_1 u - RP_1 > 0$. Therefore $K^u = \frac{V_L - \alpha P_1 u}{P_1 u - RP_1} < 0 \Longleftrightarrow V_L - \alpha P_1 u < 0 \Longleftrightarrow V_L < \alpha P_1 u$. If the bank is not allowed to default in state $\mathscr{U}$, it will be constrained, by Proposition 2, to hedging strategies $h \geqslant K^u$. Thus, short positions (i.e. $h < 0$) will only be available if $K^u < 0$, hence the statement follows. The inequality for K^d follows by the same arguments. $\quad\square$

Proof of Lemma 4. In the case where $V_L \leqslant \overline{V}$ and $\alpha P_1 d < V_L$, it follows from Lemmas 2 and 3 that $K^u \leqslant K^d < 0$. Since $V_L \leqslant \overline{V}$, there is a positive payoff to shareholders without hedging and the value of the objective function for the decision not to hedge is $I(0) = \frac{q}{B_2}(\alpha P_1 u - D_2)$. The bank is offered only hedging positions in the interval $[K^u, K^d]$ for which (by (20) of Lemma 5) the value of the objective function is $\alpha P_1 - \frac{D_2}{B_2}$ which can be written as $\frac{q}{B_2} \alpha P_1 u + \frac{1-q}{B_2} \alpha P_1 d - \frac{q D_2}{B_2} - \frac{(1-q)D_2}{B_2}$. For $h \in [K^u, K^d]$, $I(h) > I(0) \Longleftrightarrow \frac{q}{B_2} \alpha P_1 u + \frac{1-q}{B_2} \alpha P_1 d - \frac{q D_2}{B_2} - \frac{(1-q)D_2}{B_2} > \frac{q}{B_2}(\alpha P_1 u - D_2)$ which is, due to the fact that both $(1 - q) > 0$ and $B_2 > 0$, equivalent to $\alpha P_1 d \geqslant D_2$. The inequality $\alpha P_1 d < V_L$ follows from the fact that we look at the case where $K^d < 0$ and Lemma 3. $\quad\square$

Acknowledgements

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